Bayesian Probabilistic Numerical Methods

Prof. Dr. Tim Sullivan
(Free University of Berlin and Zuse Institute Berlin)

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Numerical computation, such as numerical solution of a PDE, can modelled as a statistical inverse problem in its own right. The popular Bayesian approach to inversion is considered, wherein a posterior distribution is induced over the object of interest by conditioning a prior distribution on the same finite information that would be used in a classical numerical method, thereby restricting attention to a meaningful subclass of probabilistic numerical methods distinct from classical average-case analysis and information-based complexity. The main technical consideration here is that the data are non-random and thus the standard Bayes' theorem does not hold. General conditions will be presented under which numerical methods based upon such Bayesian probabilistic foundations are well-posed, and a sequential Monte-Carlo method will be shown to provide consistent estimation of the posterior. The paradigm is extended to computational "pipelines", through which a distributional quantification of numerical error can be propagated. A sufficient condition is presented for when such propagation can be endowed with a globally coherent Bayesian interpretation, based on a novel class of probabilistic graphical models designed to represent a computational work-flow. The concepts are illustrated through explicit numerical experiments involving both linear and non-linear PDE models.